

Particle creation in flat FRW Universe in the framework of $f(T)$ gravity

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Abstract

In this paper we study the particle production in flat FRW universe in the framework of $f(T)$ gravity. Here we consider $f(T)$ theory of gravity coupled to the matter, and assuming an exact power-law solution for the field equation, we obtain a constrain on the torsion and it appears that at the late time, the finite future time singularity (big rip) can occur. The Bogoliubov coefficients are calculated for the minimally coupled massless scalar field, from which the total number of created particle per unit volume of space can be obtained. Our result shows that the particle creation rate in the framework of $f(T)$ theory of gravity is almost the same as in framework of General Relativity at early times, while, as the late time is approached, it decreases and goes to zero. We conclude that quantum effects due to particle production cannot avoid the big rip.

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1 Introduction

The possibility of particle production due to space-time curvature has been discussed by Schrodinger [1], while other early work is due to DeWitt [2], and Imamura [3]. The first thorough treatment of particle production by an external gravitational field was given by Parker [4, 5]. In flat space-time, Lorentz invariance is a guide which generally allows to identify a unique vacuum state for the theory. However, in curved space-time, we do not have Lorentz symmetry. In general, there does not exist a unique vacuum state in a curved spacetime. As a result, the concept of particles becomes ambiguous, and the problem of the physical interpretation becomes much more difficult [6, 7]. The creation of particles from the vacuum takes place due to the interaction with dynamical external constraints. For example the motion of a single reflecting boundary (mirror) can create particles [6], the creation of particles by time-dependent

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external gravitational field is another example of dynamical external constraints.

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion. Recent observations from type Ia supernovae [8] in associated with Large Scale Structure [9] and Cosmic Microwave Background anisotropies [10] have provided main evidence for this cosmic acceleration. It seems that some unknown energy components (dark energy) with negative pressure are responsible for this late-time acceleration [11]. However, understanding the nature of dark energy is one of the fundamental problems of modern theoretical cosmology. An alternative approach to accommodate dark energy is modifying the general theory of relativity on large scales. Among these theories, scalar-tensor theories [12], $f(R)$ gravity [13], DGP braneworld gravity [14] and string-inspired theories [15] are studied extensively. On the other hand, a theory of $f(T)$ gravity has recently been received attention. In [16, 17], new spherically symmetric solutions of black holes and wormholes are obtained. Models based on modified teleparallel gravity were presented, in one hand, as an alternative to inflationary models [18, 19], and on the other hand, as an alternative to dark energy models [20].

In this paper we consider the particle creation in the spatially flat Robertson-Walker space-time in the framework of $f(T)$ gravity. With the dependence of the scale factor on time, the case under consideration is a dynamical Casimir effect. We obtain the Bogoliubov coefficients, after that the number of particles produced and the energy related to those can be explicitly found.

2 Field equations with the FRW metric

Let us consider the action for the theory of modified gravity based on a modification of the teleparallel equivalent of General Relativity, namely $f(T)$ theory of gravity, coupled with matter \mathcal{L}_M , given by [20], [22] and [23, 21]

$$S = \frac{1}{16\pi G} \int d^4x e [T + f(T) + \mathcal{L}_M], \quad (1)$$

where $e = \det(e_\mu^i) = \sqrt{-g}$. The torsion T is defined as follows

$$T = S_\rho{}^{\mu\nu} T_{\mu\nu}^\rho, \quad (2)$$

where

$$T_{\mu\nu}^\rho = e_i^\rho (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i),$$

$$S_\rho{}^{\mu\nu} = \frac{1}{2} (K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\theta\nu}{}_\theta - \delta_\rho^\nu T^{\theta\mu}{}_\theta),$$

and $K^{\mu\nu}{}_\rho$ is the contorsion tensor

$$K^{\mu\nu}{}_\rho = -\frac{1}{2} (T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}).$$

The field equations are obtained by varying the action with respect to vierbein e_μ^i as follows

$$e^{-1} \partial_\mu (e S_i{}^{\mu\nu}) (1 + f_T) - e_i^\lambda T_{\mu\lambda}^\rho S_\rho{}^{\nu\mu} f_T + S_i{}^{\mu\nu} \partial_\mu (T) f_{TT} - \frac{1}{4} e_i^\nu (1 + f(T)) = 4\pi G e_i^\rho T_\rho{}^\nu, \quad (3)$$

where $f_T = f^*(T)$ and $f_{TT} = f^{**}(T)$. Now, we take the usual spatially-flat metric of Friedmann-Robertson-Walker (FRW) universe, in agreement with observations

$$ds^2 = dt^2 - a(t)^2 \sum_{i=1}^3 (dx^i)^2, \quad (4)$$

where $a(t)$ is the scale factor as a one-parameter function of the cosmological time t . Moreover, we assume the background to be a perfect fluid. Using the Friedmann-Robertson-Walker metric and the perfect fluid matter in the teleparallel Lagrangian (2) and the field equations (3), one obtains

$$T = -6H^2, \quad (5)$$

$$H^2 = \frac{8\pi G\rho}{3} - \frac{1}{6}f - 2H^2 f_T, \quad (6)$$

$$\dot{H} = -\frac{4\pi G(\rho + p)}{1 + f_T - 12H^2 f_{TT}}, \quad (7)$$

where ρ and p denote the matter density and pressure respectively, and the Hubble parameter H is defined by $H = \dot{a}/a$.

In the FRW universe, the energy conservation law can be expressed as the standard continuity equation

$$\dot{\rho} + 3H(\rho + p) = \dot{\rho} + 3H(1 + w)\rho = 0, \quad (8)$$

where ρ is the matter energy density and $p = w\rho$ is the equation of state relating pressure p with energy density. Assuming an exact power-law solution for the field equations,

$$a(t) = a_0 t^m, \quad (9)$$

where m is a positive real number, making use of the continuity equation, we obtain

$$\rho(t) = \rho_0 t^{-3m(1+w)}. \quad (10)$$

Also, using this assumption, Eq.(5) leads us to the following result

$$T = -6\frac{m^2}{t^2} < 0. \quad (11)$$

3 Particle production in expanding universe

Let ϕ denotes a scalar field of mass m , and $a(\eta)$ the scale factor for spatially flat homogeneous and isotropic FRW spacetime with conformal line element

$$ds^2 = a^2(\eta) (d\eta^2 - dx^2 - dy^2 - dz^2). \quad (12)$$

The Lagrangian density for this massive scalar field minimally coupled with the gravitational field in a conformal spacetime reads

$$\mathcal{L}_M = \frac{1}{2}a^2\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}a^4M^2\phi^2 \quad , \quad (13)$$

where $\eta_{\mu\nu}$ is Minkowski metric. The corresponding field equation reads

$$\frac{1}{a^2}\eta^{\mu\nu}\partial_\mu(a^2\partial_\nu\phi) + a^2M^2\phi = 0 \quad . \quad (14)$$

We can decompose the real scalar field into the modes as

$$\phi(\eta, \vec{x}) = \frac{1}{a} \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\vec{x}} \left[\chi_k(\eta) a_{\vec{k}} + \chi_k^*(\eta) a_{-\vec{k}}^\dagger \right] \quad . \quad (15)$$

The mode functions now satisfy the equation

$$\chi_k'' + \left(k^2 + M^2a^2 - \frac{a''}{a} \right) \chi_k = 0 \quad , \quad (16)$$

where the prime denotes the derivative with respect to the conformal time η . The modes χ_k satisfy to the Wronskian relation

$$\chi_k(\eta)\chi_k^{*\prime}(\eta) - \chi_k^*(\eta)\chi_k'(\eta) = -i \quad . \quad (17)$$

The system is quantized in a standard fashion by treating the field χ as an operator, imposing the equal-time commutation relations

$$[\chi(\eta, \vec{x}), \pi(\eta, \vec{x}')] = i\delta^3(\vec{x} - \vec{x}') \quad , \quad (18)$$

where $\pi = d\chi/d\eta \equiv \chi'$ is the canonical momentum. Then, the operators $a_{\vec{k}}$ and $a_{\vec{k}}^\dagger$ satisfy the usual commutation relations

$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = \delta^3(\vec{k} - \vec{k}') \quad , \quad [a_{\vec{k}}, a_{\vec{k}'}] = [a_{\vec{k}}^\dagger, a_{\vec{k}'}^\dagger] = 0 \quad . \quad (19)$$

The vacuum state is then defined as the state for which

$$a_{\vec{k}}|0\rangle = 0 \quad \forall k \quad . \quad (20)$$

Other states are built up from this by acting on it various combinations of creation operators $a_{\vec{k}}^\dagger$.

The Hamiltonian corresponding to (13) is

$$H = \frac{1}{2} \int d^3x \left\{ a^2 [(\phi')^2 + (\nabla\phi)^2] + a^4 M^2 \phi^2 \right\} \quad . \quad (21)$$

Making use of (15), the Hamiltonian can be re-written in terms of creation and annihilation operators and the mode functions as

$$H = \frac{1}{2} \int d^3k \left[P_k(\eta) a_{\vec{k}} a_{-\vec{k}} + P_k^*(\eta) a_{\vec{k}}^\dagger a_{-\vec{k}}^\dagger + Q_k(\eta) \left(a_{\vec{k}} a_{\vec{k}}^\dagger + a_{\vec{k}}^\dagger a_{\vec{k}} \right) \right] \quad , \quad (22)$$

where $P_k(\eta)$ and $Q_k(\eta)$ are defined by

$$P_k = \left(-\frac{a'}{a}\chi_k + \chi'_k \right)^2 + \omega_k^2 \chi_k^2 \quad , \quad (23)$$

$$Q_k = \left(-\frac{a'}{a}\chi_k + \chi'_k \right) \left(-\frac{a'}{a}\chi_k^* + \chi_k^{*\prime} \right) + \omega_k^2 \chi_k \chi_k^* \quad , \quad (24)$$

with $\omega_k^2 = k^2 + M^2 a^2$.

In general, the field may be decomposed into many different complete set of modes and each of these has its own vacuum state. Suppose we label one such set of modes as $\bar{\chi}(\eta)$. Then, in terms of these modes the field is

$$\phi(\eta, \vec{x}) = \frac{1}{a} \int \frac{d^3 k}{(2\pi)^{3/2}} e^{-i\vec{k}\vec{x}} \left[\bar{\chi}_k(\eta) \bar{a}_{\vec{k}} + \bar{\chi}_k^*(\eta) \bar{a}_{-\vec{k}}^\dagger \right] \quad , \quad (25)$$

and the vacuum state is defined by $\bar{a}_{\vec{k}} |0\rangle = 0$ for all \vec{k} . The Hamiltonian in this case is written as

$$H = \frac{1}{2} \int d^3 k \left[\bar{P}_k(\eta) \bar{a}_{\vec{k}} \bar{a}_{-\vec{k}} + \bar{P}_k^*(\eta) \bar{a}_{\vec{k}}^\dagger \bar{a}_{-\vec{k}}^\dagger + \bar{Q}_k(\eta) \left(\bar{a}_{\vec{k}} \bar{a}_{\vec{k}}^\dagger + \bar{a}_{\vec{k}}^\dagger \bar{a}_{\vec{k}} \right) \right] \quad , \quad (26)$$

where \bar{P}_k and \bar{Q}_k are similar expressions as in (23) and (24) respectively, replacing χ_k by $\bar{\chi}_k$. Because of completeness, the two sets of modes χ_k and $\bar{\chi}_k$ are related by the Bogolubov transformations that diagonalize the Hamiltonian i.e., $\bar{P}_k(\eta) = 0$, satisfying the relation

$$\bar{\chi}_k(\eta) = \gamma_k \chi_k(\eta) + \beta_k \chi_k^*(\eta) \quad , \quad (27)$$

with the normalization condition $|\gamma_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1$, where γ_k and β_k are constants and called Bogolubov coefficients. One can compare the two vacuum states by noting that the number operator for the barred states is $\bar{N} \equiv \int d^3 k \bar{a}_{\vec{k}}^\dagger \bar{a}_{\vec{k}}$. Taking its expectation value with respect to the unbarred vacuum, one finds

$$\langle 0 | \bar{N} | 0 \rangle = \int d^3 k |\beta_k|^2 \quad . \quad (28)$$

Thus, the number of barred particles in the unbarred vacuum in the mode \vec{k} is $|\beta_k|^2$. Similarly, the number of unbarred particles in the barred vacuum in the mode \vec{k} is $|\beta_k|^2$. Since the diagonalization imposes $\bar{P}_k(\eta) = 0$, using (23) with χ_k replaced by $\bar{\chi}_k$, we get

$$-\frac{a'}{a} \bar{\chi}_k + \bar{\chi}'_k = -i\omega_k \bar{\chi}_k \quad . \quad (29)$$

Substituting this result in the expression of $\bar{Q}_k(\eta)$, we get

$$\bar{Q}_k(\eta) = 2\omega_k^2(\eta) |\bar{\chi}_k|^2 \quad . \quad (30)$$

Using (22) and (26) we obtain

$$\langle 0 | 2\bar{a}_{\vec{k}}^\dagger(\eta) \bar{a}_{\vec{k}}(\eta) + 1 | 0 \rangle = \frac{Q_k(\eta)}{\bar{Q}_k(\eta)} \quad , \quad (31)$$

and consequently

$$|\beta_k(\eta)|^2 = \frac{1}{2} \frac{Q_k(\eta)}{\bar{Q}_k(\eta)} - \frac{1}{2} . \quad (32)$$

Using now (24) and (30) one can rewrite (32) as

$$|\beta_k(\eta)|^2 = \frac{1}{4} \frac{\left[\left(\frac{a'}{a} \right)^2 + \omega_k^2(\eta) \right] |\chi_k|^2 - \frac{a'}{a} (\chi_k \chi_k^{*'} + \chi_k' \chi_k^*) + |\chi_k'|^2}{\omega_k^2(\eta) |\bar{\chi}_k|^2} - \frac{1}{2} . \quad (33)$$

There are two important point to be understood with respect to vacuum states in curved space. The first is that, in general, it is uncertain what criteria should be used in choosing the vacuum state. The problem is that many of criteria used for Minkowski space as Lorentz invariance and positive frequency with respect to a timelike Killing vector no longer apply in curved space. The second one is that, if there is some "natural" choice of vacuum when the spacetime begins, it does not in general correspond to the natural choice of vacuum when the spacetime ends. That is, the "in" vacuum state and the "out" vacuum state are different. This leads to particle production as can be seen from (33).

Now, to calculate $|\beta_k(\eta)|^2$ we have to find $\chi_k(\eta)$ from (16) and $\bar{\chi}_k(\eta)$ from (29). The factor scalar corresponding to (9) in terms of the conformal time is

$$a(\eta) = \frac{B}{(-\eta)^{\frac{m}{m-1}}} , \quad B = [a_0(m-1)^m]^{\frac{1}{1-m}} , \quad -\infty < \eta < 0 . \quad (34)$$

For guaranteeing an expanding and accelerated universe, the parameter m is constrained to the condition $m > 1$.

Making use of (34) in (16), the general solution can be written in terms of Hankel functions as

$$\chi_k(\eta) = \frac{\sqrt{\pi|\eta|}}{2} \left[A_k H_\nu^{(1)}(k|\eta|) + B_k H_\nu^{(2)}(k|\eta|) \right] , \quad \nu = \sqrt{\frac{1}{4} + \frac{m(2m-1)}{(m-1)^2}} , \quad (35)$$

where A_k and B_k are constants to be determined. Making use of the Wronskian relation

$$z H_\nu^{(2)}(z) \partial_z H_\nu^{(1)}(z) - z H_\nu^{(1)}(z) \partial_z H_\nu^{(2)}(z) = \frac{4i}{\pi} , \quad (36)$$

one obtains, imposing orthonormalization of the modes,

$$|B_k|^2 - |A_k|^2 = 1 . \quad (37)$$

For fixing the initial vacuum state we use the Bunch-Davies state [24, 25] by the choice $A_k = 0$ and $B_k = 1$, and the solution (35) becomes

$$\chi_k(\eta) = \frac{\sqrt{\pi|\eta|}}{2} H_\nu^{(2)}(k|\eta|) . \quad (38)$$

Note that the solution (38) reduces to the Minkowski one in which particle production phenomenon does not occur.

Now we can proceed to the calculation of $|\beta_k(\eta)|^2$ through the expression (33). Since we are dealing with a minimally coupled massless scalar field, the expression (33) becomes

$$|\beta_k(\eta)|^2 = \frac{1}{4} \frac{\left(\frac{q^2}{\eta^2} + k^2\right) |\chi_k|^2 - \frac{q}{\eta} (\chi_k \chi_k^{*'} + \chi_k' \chi_k^*) + |\chi_k'|^2}{k^2 |\bar{\chi}_k|^2} - \frac{1}{2}, \quad q = \frac{m}{1-m} \quad (39)$$

Now, substituting (38) into (39), we get

$$\begin{aligned} |\beta_k(\eta)|^2 = & \frac{\pi}{8k} \left\{ - \left[\frac{(q-1/2)^2}{\eta} + k^2 \eta \right] H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta) \right. \\ & - \frac{k}{2} (q-1/2) \left[H_\nu^{(1)}(-k\eta) H_{\nu-1}^{(2)}(-k\eta) + H_\nu^{(2)}(-k\eta) H_{\nu-1}^{(1)}(-k\eta) \right. \\ & \quad \left. \left. - H_\nu^{(1)}(-k\eta) H_{\nu+1}^{(2)}(-k\eta) - H_\nu^{(2)}(-k\eta) H_{\nu+1}^{(1)}(-k\eta) \right] \right. \\ & - \frac{k^2 \eta}{4} \left[H_{\nu-1}^{(1)}(-k\eta) H_{\nu-1}^{(2)}(-k\eta) - H_{\nu-1}^{(1)}(-k\eta) H_{\nu+1}^{(2)}(-k\eta) \right. \\ & \quad \left. \left. - H_{\nu+1}^{(1)}(-k\eta) H_{\nu-1}^{(2)}(-k\eta) + H_{\nu+1}^{(1)}(-k\eta) H_{\nu+1}^{(2)}(-k\eta) \right] - \frac{4k}{\pi} \right\} \quad (40) \end{aligned}$$

Note that at the early time, $\eta \rightarrow -\infty$, $H_\nu^{(2)}(-k\eta) \sim \sqrt{-\frac{2}{k\pi\eta}} e^{i(k\eta + \frac{\pi}{4} + \frac{\nu\pi}{2})}$ and using (40), we obtain $|\beta_k(-\infty)|^2 = 0$. We see clearly through (40) that particle production can be known at any time and also the initial vacuum condition is correctly reproduced at early time where there is no particle production. Then, as the conformal time grows, particle creation becomes important. The total number $N(\eta)$ of created particle per unit of space volume is [26]

$$\begin{aligned} N(\eta) &= \int N_k(\eta) \\ &= \frac{1}{2\pi^2 a^3(\eta)} \int k^2 dk |\beta_k(\eta)|^2 \quad (41) \end{aligned}$$

In terms of the conformal time, the torsion can be written as

$$T = - \frac{6m^2}{[a_0(1-m)\eta]^{\frac{2}{1-m}}} \quad (42)$$

This means that at the early times ($\eta \rightarrow -\infty$), the torsion scalar $T \rightarrow -\infty$, while as $\eta \rightarrow 0$, the torsion scalar vanishes. This situation is quite different to the case in which the gravitational part of the action is driven by the curvature. In General Relativity (GR) the particle production is directly connected with the curvature of the universe. When the curvature goes to zero or almost null, there is no particle production, while as the curvature increases, the particle production becomes important. Here, it appears that in $f(T)$ gravity, the vacuum is reproduced when the torsion scalar diverge, at least in the FRW universe. As the torsion scalar approaches zero, the particle production becomes important (see Fig1).

On the other hand it is easy to note that as the $\eta \rightarrow 0$, the scale factor, the energy density and the pressure diverge; this is a big rip (singularity of type I). For the classification of the future finite time

singularities, see [27]. It is quite natural in such a situation to investigate quantum effects due to particle as the singularity time is approached. The evolution of the rate of particle production for some modes, as the universe evolves, are given in Fig.1.

From the Fig1 we see that for each mode, the density of created particle is initially important. As the conformal time evolves this quantity grows and reach a maximum value, and after, starts decreasing and goes to zero at the singularity time. In other word, this means that there is almost no particle production near the singularity. Thus, the energy density associated to particle production vanishes consequently at the singularity time. However, as the singularity time approaches, the classical energy density of the background diverges. In such situation, the big rip is robust against quantum effects due to particle creation. This results perfectly agrees with that obtained in studying quantum effect due to particle against the finite time singularities in the framework of GR and $f(R)$ modified gravity. Thus, in the framework of GR, it is shown that quantum effects cannot avoid the occurrence of the big rip [24, 31, 29, 30]. Also, in the framework of GR, the same result is obtained about the sudden singularity, showing the non-efficiency of quantum effect against this sort of finite time singularity [28, 27]. In the framework of $f(R)$ gravity, the weakness of quantum effects is also confirmed against the big rip[32].

4 Conclusion

In this paper we have studied the particle creation in FRW universe in the framework of $f(T)$ gravity. Since the case when the spatial sections are flat is slightly simpler, hence we have restricted to that case here. We have assumed an exact power-law solution for the scale factor of universe, which leads us to a constrained torsion for the universe. The scale factor in terms of the conformal time is given by Eq.(34). For guaranteeing an accelerated expanding universe the parameter m is constrained to the condition $m > 1$. For fixing the initial vacuum state we have considered the Bunch-Davies state and have obtained the mode function as Eq.(38). Then we have obtained $|\beta_k(\eta)|^2$ for the minimally coupled massless scalar field as Eq.(40). One can see from this expression that in the early time, $\eta \rightarrow -\infty$, $|\beta_k(-\infty)|^2=0$, but in this case $T \rightarrow -\infty$. So we have shown that in the framework of $f(T)$ gravity, when the torsion is divergence, the particle creation rate is zero. It is in contrast with the result in the framework of general relativity, where the curvature increase, the particle creation becomes important. Our results have shown that there is almost no particle production near the singularity. So the energy density associated to particle production vanishes at the singularity time, when the classical energy density of the background diverges. Therefore the big rip is robust against quantum effects due to particle creation.

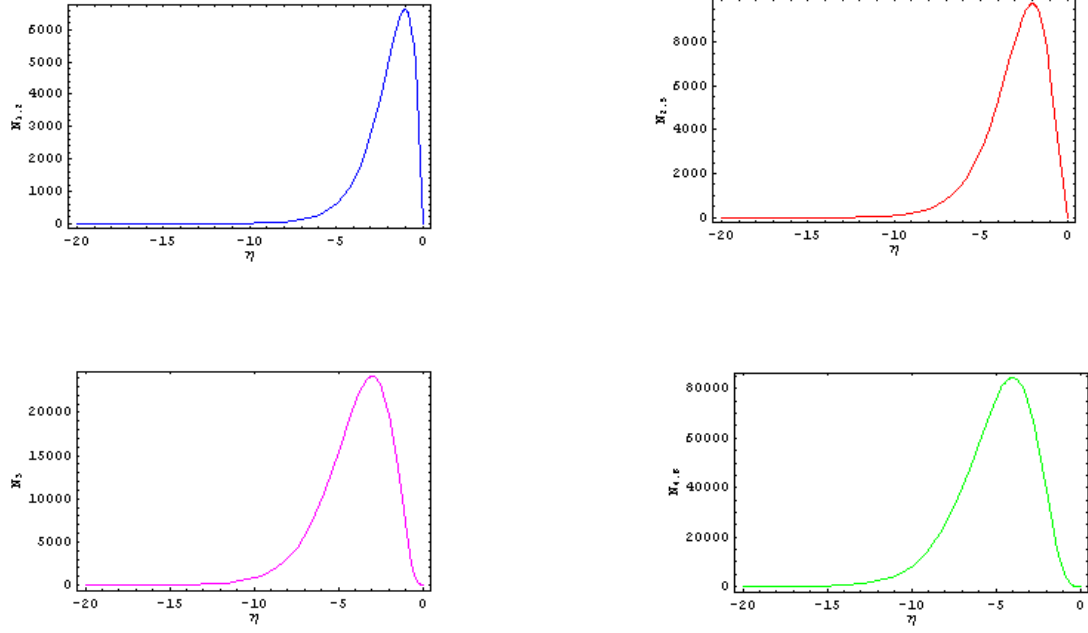


Figure 1: These figures show the evolution of the density of created particle N_k for four different values of the mode k with the parameter $m = 2$. The blue (upper left) corresponds to $k = 1.2$, the red (upper right) corresponds to $k = 2.5$, the magenta (lower left) corresponds to $k = 3$ and the green (lower right) corresponds to $k = 4.6$.

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